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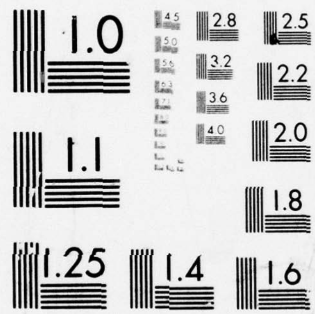
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NSWC/DL TR-3535

OCEAN TIDES IN THE EQUATIONS OF MOTION OF THE TERRA SYSTEM OF COMPUTER PROGRAMS FOR SATELLITE GEODESY

by
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DECEMBER 1976

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An algorithm is specified for the Newtonian attraction which the ocean tide exerts on artificial satellites of the earth. Only the semidiurnal lunar tide is presently considered. But, it is indicated how various other spectral components of the fluid tide may be accounted for, if desired.

The algorithm starts from a table for the global tide amplitudes and phase angles. It proceeds by approximating the gravitational potential exterior to the earth, caused by the

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tidal bulge, by that of a system of point masses on the ocean surface. For the sake of flexibility, separate computational schemes are described for the potential and the perturbing acceleration, the former being independent of the latter. This is to facilitate investigations which may eventually be conducted, such as studies of the shape of the tidal bulge.

FOREWORD

This report documents another of the perturbing terms appearing in the force equations for satellite motion in the TERRA system of satellite geodesy computer programs. A detailed algorithm is presented for the gravitational action which the oceanic tide bulge exerts on a satellite. Two earlier reports described the necessary equations for the air tides caused by sun and moon. Also, the manuscript has been completed for a further report which will present an algorithm for the perturbing force caused by the tidal deformation of the solid earth.

The following text was reviewed and approved by Mr. R. J. Anderle, Head Astronautics and Geodesy Division.

Released by:

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INTRODUCTION

TERRA[1] is a new system of computer programs for satellite geodesy.[2] Like its forerunners, it performs geodetic solutions for a variety of parameters such as the constants of the gravity field model, the observing station coordinates, the orbital parameters of the satellites involved and certain of the geophysical and astronomical constants which determine the satellite orbits. To be able to take advantage of the recent progress in instrumentation for data acquisition, TERRA is a collection of algorithms some of which are rather more complex than the corresponding schemes from its forerunners. In particular, the force equations for satellite motion now contain terms which quite realistically indicate the effect of certain physical phenomena which are formerly modelled in a crude fashion only or which were discounted altogether.

Amongst the forces previously omitted but included in TERRA, those stand out which reflect the gravitational action of the various types of earth tide on the satellite orbits. They are the air tides caused by the moon and the sun, the ocean tide, and the tide of the solid earth body. Equations for the two atmospheric tides were elaborated on recently.[3,4] Also, the solid earth tide is already a part of the TERRA coding. It is the subject of a technical report.[5]

For the solid earth tide, we obtained a potential from the literature in a form immediately applicable to our purpose. Only the gradient needed to be found. That involved a large computing effort which was in the end accomplished with the help of an analytical computer language. For the two air tides, surface pressure functions corresponding to the atmospheric tide bulges were readily available in the geophysical literature. From surface pressure, we managed to derive the disturbing acceleration for each tide via Poisson integration followed by spatial differentiation.

The present report on the ocean tide in TERRA is the third in a series of four, each presenting one of the four individual tides. Actually, the ocean tide was the last for which we succeeded in our effort to produce an adequate, yet manageable, disturbing term. For this there are reasons inherent in the problem. There appears to be a particular aspect to the ocean tides which sets them apart from the remaining earth tides. To be specific, any effort to mathematically formulate the shape of the tidal bulge for any of the four tides may be expected to involve some kind of eigenfunction expansion. The same is true for any detailed description of items derived from the tidal bulge, such as Newtonian potentials arising in connection with the tidal mass redistribution and the associated gradients. For the solid earth tides and the air tides, these expansions are meaningful on a

domain which is the entire earth surface and/or the space above it. The dominant boundary condition is simple, merely requiring continuity and periodicity in longitude. This reflects the fact that there exists no sharply pronounced barrier to tidal wave propagation in the atmosphere; neither is there such a barrier within the solid earth. But, barriers of this type do occur in the ocean. They are the coastlines. That fact gives rise to a good deal of mathematical unpleasantness. For example, if one desires to represent the tidal ocean surface by the customary surface harmonic expansion, one will have to decide whether the mathematical expression for that surface is to be valid for any possible pair of values for latitude and longitude or if one wishes to produce an expression which is valid only for a domain which coincides with the ocean. The former expression will inherently be capable of yielding zero tidal deflection for any point located inland, within a specified threshold. But it obviously will have to contain a formidable number of parameters. The latter expansion will be of the type which the physicist calls "non-analytic." It will be simpler than the first, but it will be valid only on the ocean domain. Thus, we shall have to augment it by the familiar postulate that it be disregarded on that domain which consists of land and that its value there be zero by definition.

We did indeed encounter the latter situation when we tried to adapt Hendershott's expansion for the ocean tide surface[6] to our problem. On a superficial inspection, Hendershott's equation for the tidal surface appeared well-suited for conversion to a tidal mass layer, as done successfully for the air tides in References 3 and 4. But, when we subsequently tried to actually formulate and execute the Poisson integral to calculate the tide potential exterior to the earth as we had done for the air tides, we immediately faced the task of spelling out the integration boundary. This task looked difficult because Hendershott's equation appeared to be of what we called the "non-analytical" type above. Consequently, we had the option of trying to develop integration boundaries which would reflect the continental coastlines. Or else, we might attempt to achieve very simple integration boundaries by converting the formula for the tidal surface into one which would be valid on the entire globe, yielding spurious values of negligible magnitude on land. Because of the great complexity inherent in either approach, we abandoned the Hendershott model. This decision was made easy by the circumstance that the number of expansion coefficients listed in Reference 6 was clearly insufficient for our purpose. Further, our reference indicated that an additional number of these coefficients had actually been computed. But we were unable to obtain those from the author.

In fact, we abandoned not only the Hendershott model, but we resolved to entirely stay clear of concepts like continuous surface densities and Poisson integrals. We were, of course, aware of the important work done in the field of earth tide effects on satellite motion which is being published under the names of K. Lambeck, A. Cazenave, and G. Balmino.[7,8,9] We hesitated to get involved with these papers because they represent continuing research. They certainly identify various significant components of the tide potential. We desired, however, to specify, for use with TERRA, a tide potential which would implicitly contain not only the known tidal effects but, additionally, as many of those perturbations which might be characteristic of any particular type of satellite orbit to which TERRA might be applied.

Fortunately, an entirely different approach offered itself. A new theory for the ocean tide was recently developed at the Naval Surface Weapons Center, Dahlgren Laboratory (NSWC/DL), by Dr. E. Schwiderski. This is an improved Zahel theory.* A computer model which implements this theory has just begun to yield results which are in excellent agreement with the observed data. At the present time, this model reflects only the M_2 component of the tide. The latter is thought to account for a substantial portion (about 70 percent) of the oceanic mass dislocation due to the tides. Also, it appears that it is not a technical problem but purely a question of funding to generalize this model to include any desired member of the tide spectrum for which sufficient observations are available. As the present version appears by itself to be the most accurate and detailed one amongst the quantitative descriptions known for the ocean tide, we confidently based our work on it.

The Schwiderski model presents the tidal ocean surface as a listing of values, for the tidal amplitudes and phase angles, at the center points of surface area elements which form a grid covering most of the oceans. This permitted us to specify two algorithms which, if used in sequence, should enable TERRA to account for the orbital perturbations due to the tide bulge.

In detail, we regard the tidal height on each ocean surface element, as computed by the Schwiderski model, as a measure for that portion of the mass of the tidal bulge which is located within the particular area element. We postulate that each tidal height value thus resembles a gravitating point mass which perturbs the satellite by its presence. The gravitational potential for each point mass is then expanded in terms of Legendre functions. Upon further summation, over all area elements, the resulting perturbing potential is decomposed into products of terms which contain the source point coordinates and other terms which are functions of

*For details on the Zahel theory, see References 10 and 11.

the field point (satellite position) and time only. The latter terms act the part of eigenfunctions while the former are the expansion coefficients of the perturbing potential.

The quantity actually needed is the perturbing acceleration which is the gradient of the perturbing potential. This can be easily assembled from the just mentioned expansion coefficients and eigenfunctions because, as any reasonably complete text on potential theory or related subjects will show, it is possible to express the spatial derivatives of the solid harmonics occurring in our potential as linear combinations of certain of the solid harmonics themselves.

The resulting scheme for computing the perturbing acceleration quite closely resembles that used to evaluate the spherical harmonics occurring in that part of the TERRA equations of motion which deals with the main part of the earth's gravitational field. Every effort was made while constructing the two algorithms to enable the programmer to utilize relevant procedures for which codings already exist such as the recurrence relationships for the spherical harmonics. The latter are used in a number of computer programs for satellite geodesy[12] because they greatly facilitate the computation of the eigenfunctions occurring in the expressions for the geodetic potentials, and thus, in the perturbing accelerations which act on the satellites as a consequence of the presence of these potentials.

What might conveniently have been one single collection of equations for coding was partitioned into two separate algorithms, for various reasons. Work done in the past[12] had demonstrated the power and convenience of our recursive schemes for the eigenfunctions of the potential. These codings require that the constituents of the potential, namely the eigenfunctions and the expansion coefficients, be known before the field gradient may be evaluated. Finding the expansion coefficients and the eigenfunction values is the tedious part of the task. But, once they are known, the the corresponding gradient vector follows easily. This suggested that we relegate the two just mentioned operations to a preprocessor algorithm which feeds data into a second algorithm which is part of the main body of TERRA and which constitutes the perturbing acceleration associated with the tide. Also, a second motive for dividing the procedure was that we anticipated that we might wish to study certain aspects of the tide potential which are rather unrelated to satellite motion in TERRA. One obvious application would be to find out how well the original tide bulge can be resynthesized from the potential generated by the above mentioned point masses. Investigations of this type would be greatly aided if it were possible to isolate that coding which produces the ocean tide perturbing potential from the body of TERRA.

As will be seen below, this separation was, by necessity, not a perfect one. But, it provides for a preprocessor which contains most of the individual features of the tide model in use, such as the parameters of the tidal point masses. At the same time it permits the actual perturbing term to appear in the equations of motion in a form which is quite independent of the individual properties of the tide model.

More precisely, the first of our algorithms stands alone, by itself, outside the body of computer programs which form TERRA. Its task is to find, from the tidal distortion of the ocean surface, the numerical values of the tidal point masses as well as their positions. From here, it calculates the expansion coefficients for the perturbing potential. Also, it is capable of calculating values for the eigenfunctions of the potential, for any given field point.

Along with items like the tidal frequency and related astronomical data (specifying the position of the celestial body or bodies causing the tide), the just mentioned expansion coefficients are the input for the second routine. The latter is an integral part of the TERRA equations of motion. Like the first algorithm, it is equipped to evaluate the eigenfunctions (which are now to be regarded "eigenfunctions of the perturbing potential"). The results of that procedure are subsequently combined with the expansion coefficients to form the spatial components of the perturbing acceleration for the satellite motion. Note that the separation of the two algorithms is imperfect because there appear certain multipliers in both of them which reflect the time-dependent nature of the tide. In the form in which they appear below, these multipliers are valid for the M_2 tide only. Caution will have to be exercised to adapt them to any different tide which may be introduced later.

The first algorithm will be executed infrequently. As the Schwiderski model specifies its results in terms of discrete values at grid points which are 1 degree apart in longitude and latitude, our first algorithm utilizes about 45 000 tidal amplitudes and phases to compute 45 000 complex numbers (number pairs) representing point masses which are, in turn, converted to 45 000 pairs of complex spherical harmonic coefficients. One or two dozen of these pairs of complex spherical harmonic coefficients will be hand-selected for input to CELEST/TERRA which will be modified to operate on these coefficients with the second algorithm. Experiments will then be conducted to determine the minimum number of coefficients needed to reflect the action of the ocean tides on the satellite orbits.

TIDAL POINT MASSES

To find the tidal point masses and their positions, divide that portion of the globe which is ocean into area elements, according to the output format of the Schwiderski Ocean Tide Program. For details, see Figures 1 and 2. Note that each surface area element has sides the angular length of which is 1 degree. Exclude from consideration all those area elements which the Ocean Tide Program discounts, especially those located over land and all those which are located below a certain southern latitude. These area elements which contain the North pole are, of course, triangles. For the purpose of mass point location, assume that the area elements be located on the reference ellipsoid. As far as area size is concerned, assume that the area elements be located on a sphere of radius R. Most frequently, R will be equated to the semimajor axis of the reference ellipsoid. Specify R in terms of kilometers.

Now, place a point mass, m_{ij}, into the center of all "valid" area elements, as indicated in Figure 2. Number all surface area elements, point masses and associated quantities either by a double subscript, ij, or index, ν, as expedient. Generally, regard ij and ν as interchangeable indices. Figures 1, 2, and 3 now suggest how to form the expressions for the surface areas of the area elements,

$$\Delta S_\nu = \left(\frac{\pi}{180}\right)^2 R^2 \sin\left(\frac{\pi}{180} j\right) \tag{1}$$

$$j = 2,3,4, \cdots, j_{max} < 180 ,$$

and

$$\Delta S_{\nu P} = \frac{1}{2}\left(\frac{\pi}{180}\right)^3 R^2 \tag{2}$$

which is the surface area of the "polar" surface element.

Specify the positions of the point masses as follows.

$$\vartheta_j = \frac{\pi}{180} j \tag{3}$$

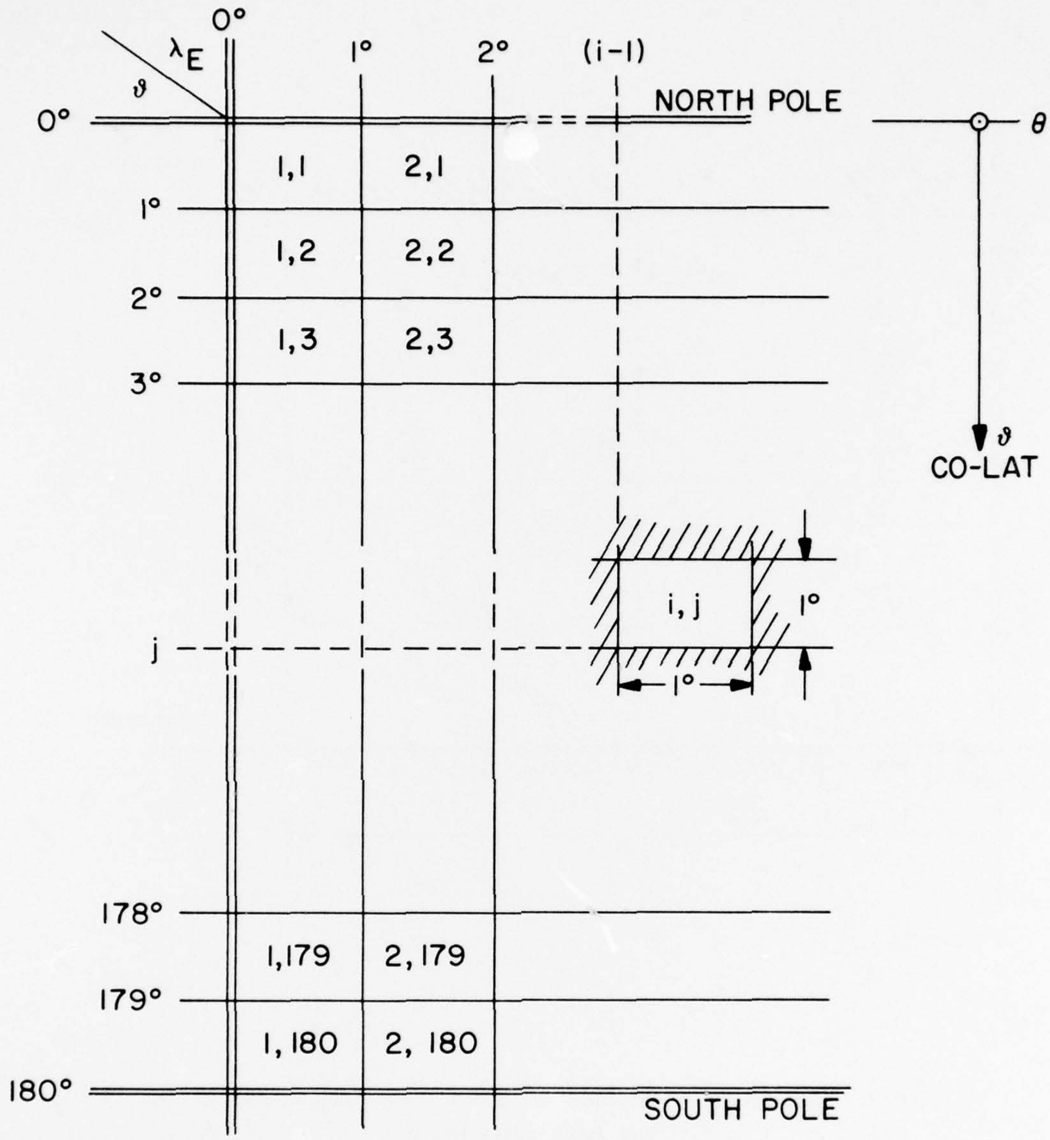

Figure 1. Division of the Earth's Surface into Area Elements According to the Schwiderski Ocean Tide Model

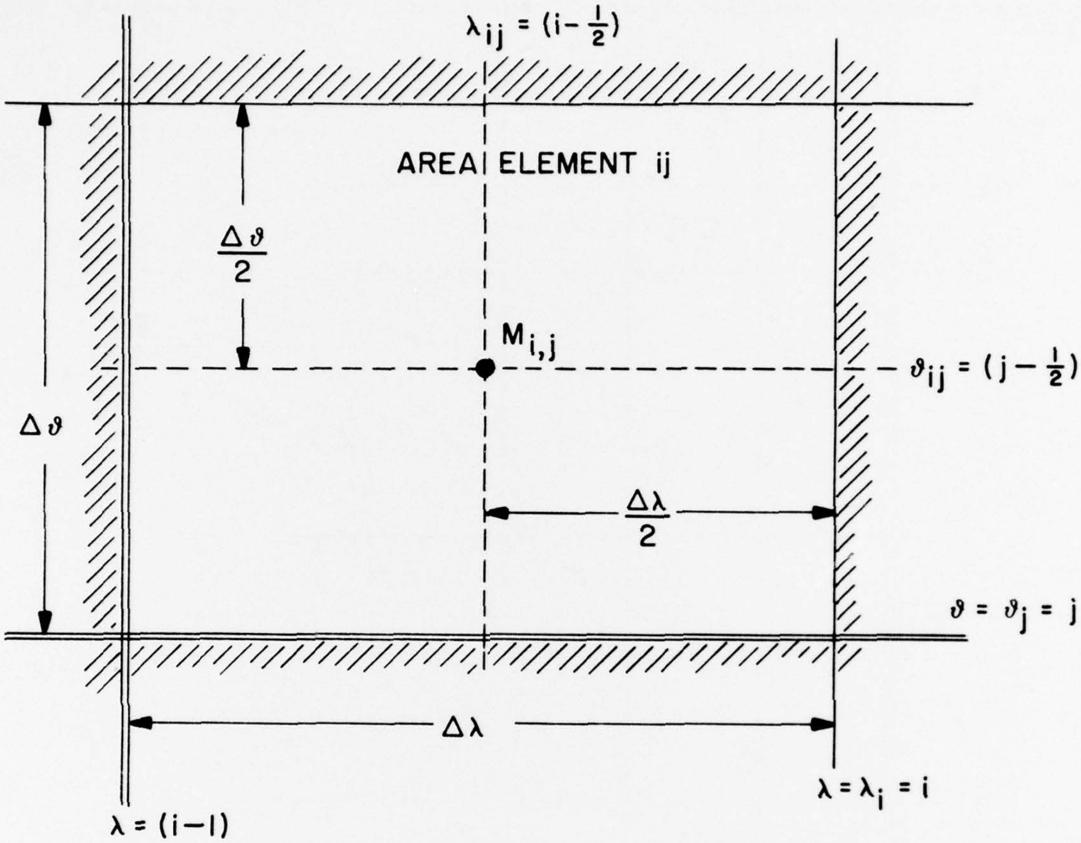

Figure 2. Position of Point Mass at the Geometrical Center of the Surface Area Element, ij

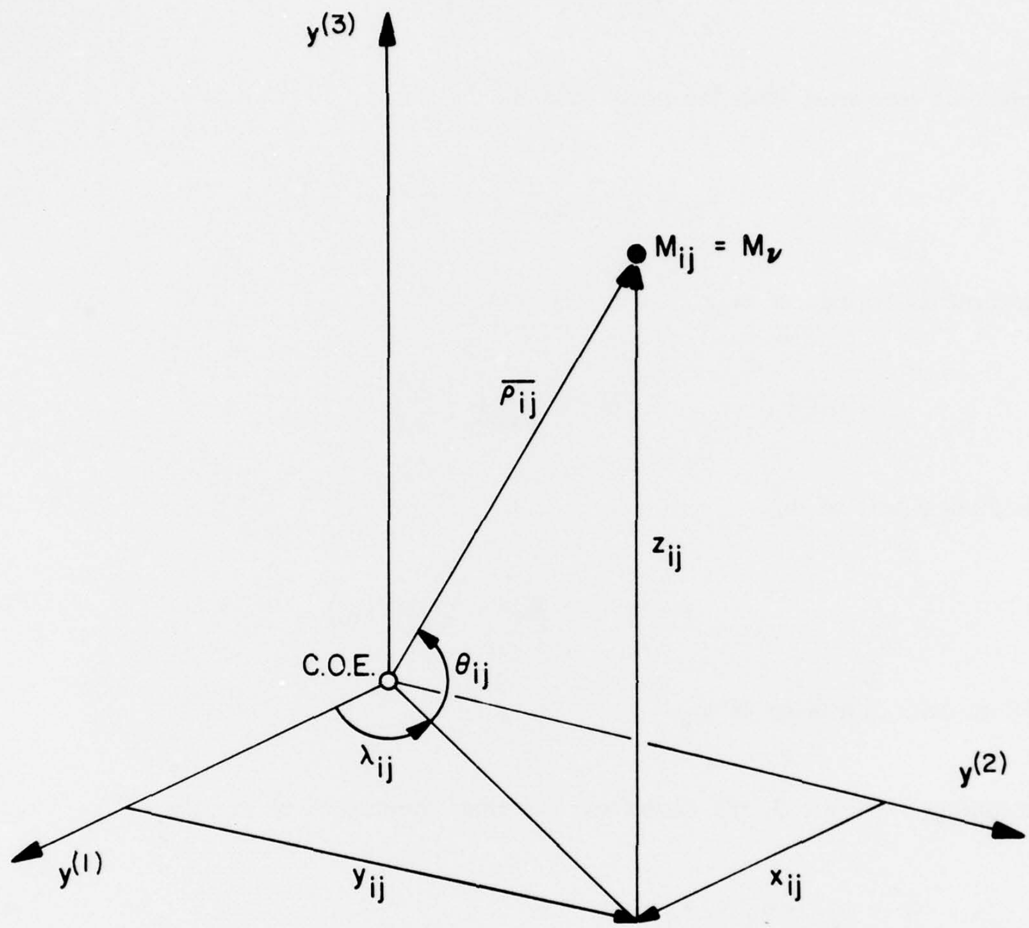

Figure 3. Position of the Point Mass in the Earth-Fixed Cartesian Coordinate Frame $y^{(1)}$, $y^{(2)}$, and $y^{(3)}$ [where $y^{(1)}$ is in the Direction of the Point where Greenwich Meridian and Equator Intersect and $y^{(2)}$ is in the Equatorial Plane]

the colatitudes associated with the area element, ij.

$$\vartheta_{ij} = \vartheta_\nu = \frac{\pi}{180}\left(j - \frac{1}{2}\right) \tag{4}$$

colatitude associated with the point mass, ij.

$$\theta_{ij} = \frac{\pi}{2} - \vartheta_{ij} = \frac{\pi}{2} - \frac{\pi}{180}\left(j - \frac{1}{2}\right) \tag{5}$$

(geocentric) latitude of m_{ij}.

$$\lambda_{ij} = \frac{\pi}{180}\left(i - \frac{1}{2}\right) \tag{6}$$

longitude (east) of m_{ij}.

$$\rho_{ij} = \rho_\nu = R\left(1 - \frac{\epsilon^2}{2}\sin^2\theta_{ij}\right) \tag{7}$$

the geocentric distance of m_{ij}.

According to Figure 3, the earth-fixed Cartesian coordinates of m_{ij} are

$$x_{ij} = \rho_{ij}\cos\theta_{ij}\cos\lambda_{ij} \tag{8}$$
$$y_{ij} = \rho_{ij}\cos\theta_{ij}\sin\lambda_{ij} \tag{9}$$
$$z_{ij} = \rho_{ij}\sin\theta_{ij} \tag{10}$$

Above, ϵ^2 is the square of eccentricity of the reference ellipsoid. In case it is desired to start from the flattening, f, of the ellipsoid, find ϵ^2 from

$$\epsilon^2 = (2 - f)f \tag{11}$$

Further, ρ is the density of sea water and G is Newton's constant of mass attraction. Assume

$$\rho = 1 \tag{12}$$

for a sea water density of 1 g/cm^3. If one wishes to employ a different density value, scale ρ accordingly. Also,

$$G = (6.67\,E-20)\ \text{km}^3/\text{kg sec}^2 \tag{13}$$

Now, let ζ_ν be the tidal amplitude, in meters, on area element, ij. δ_ν is the associated tidal phase angle. Both constitute printout from the Ocean Tide Program. t* is Universal time, in seconds (reference zero is midnight UT). σ is the circular frequency associated with the M_2 component of the ocean tide (two cycles per lunar mean day).

χ is the mean mean longitude of the moon, in degrees, at the beginning of the particular day UT. A routine is indicated for χ, below. And

$$\sigma = \frac{180}{\pi}(1.40519E-04)\ \text{sec}^{-1} \tag{14}$$

Then, define the "gravitational charge" (capable of assuming positive as well as negative values) of the point mass:

$$m_\nu = \frac{1}{G}[\alpha_\nu \cos(\sigma t^* + \chi) + \beta_\nu \sin(\sigma t^* + \chi)] \tag{15}$$

$$\alpha_\nu = 10^9\,\rho\,G\,\Delta S_\nu\,\zeta_\nu \cos\delta_\nu \tag{16}$$

$$\beta_\nu = 10^9\,\rho\,G\,\Delta S_\nu\,\zeta_\nu \sin\delta_\nu \tag{17}$$

The latter two quantities are expected to result in terms of km^3/sec^2. Thus, the unit for m_ν will be kilograms.

Let N be the number of point masses. We expect N to roughly equal 45 000.

To find χ, execute the following routine. Let J be the number of the calendar year (1975 or 1976 or 1977; generalization to other years is obvious), M be the number of the individual month in the calendar year, and D be the number of the particular day in the month. Also, let

$$\delta(n,m) = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases} \tag{18}$$

Then

$$\begin{aligned} N_1 = {} & D + 31\delta(M,2) + 59\delta(M,3) + 90\delta(M,4) + 120\delta(M,5) + 151\delta(M,6) \\ & + 181\delta(M,7) + 212\delta(M,8) + 243\delta(M,9) + 273\delta(M,10) \\ & + 304\delta(M,11) + 334\delta(M,12) \end{aligned} \tag{19}$$

$$\begin{aligned} N_2 = {} & D + 31\delta(M,2) + 60\delta(M,3) + 91\delta(M,4) + 121\delta(M,5) + 152\delta(M,6) \\ & + 182\delta(M,7) + 213\delta(M,8) + 244\delta(M,9) + 274\delta(M,10) \\ & + 305\delta(M,11) + 335\delta(M,12) \end{aligned} \tag{20}$$

$$N_\Sigma = N_1\,\delta(J,1975) + (365 + N_2)\delta(J,1976) + (731 + N_1)\delta(J,1977) \tag{21}$$

$$\Delta T = (5.28E - 4) + (3.56E - 8)N_\Sigma \tag{22}$$

$$d_0 = 27392.5 + N_\Sigma + \Delta T \tag{23}$$

$$T_0 = \frac{d_0}{36525} \tag{24}$$

$$\chi = 270.434358 + 481267.88314137\, T_0 - 0.001133\, T_0^2 + 0.0000019\, T_0^3 \tag{25}$$

EXPANSION COEFFICIENTS FOR THE TIDE POTENTIAL

We shall now indicate how the expansion coefficients can be calculated for the disturbing potential exterior to the earth, produced by the combined action of the tidal point masses. This potential has the general form

$$\phi = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} (F_{nm} U_{nm} + H_{nm} V_{nm}) \tag{26}$$

F_{nm} and H_{nm} are the expansion coefficients. U_{nm} and V_{nm} are the eigenfunctions.* Note that the coefficients of the expansion can be expected to depend on time, because the point masses from which they result oscillate with the tide. The eigenfunctions are solid harmonics which in turn are functions of the field point coordinates (satellite position vector).

Obtain the necessary input data from the previous chapter. Note that μ is the earth's gravitational constant. Select for it a value compatible with the rest of the TERRA program. Express μ in terms of kilometers and seconds. Find now

$$F_{nm} = \alpha_{F_{nm}} \cos(\sigma t^* + \chi) + \beta_{F_{nm}} \sin(\sigma t^* + \chi) \tag{27}$$

$$H_{nm} = \alpha_{H_{nm}} \cos(\sigma t^* + \chi) + \beta_{H_{nm}} \sin(\sigma t^* + \chi) \tag{28}$$

$$\alpha_{F_{nm}} = \left(2 - \delta_m^0\right) \frac{(n-m)!}{(n+m)!} \frac{1}{\mu R^{2n}} \sum_{\nu=1}^{N} \rho_\nu^{2n+1} \alpha_\nu f_{nm}^\nu \tag{29}$$

$$\beta_{F_{nm}} = \left(2 - \delta_m^0\right) \frac{(n-m)!}{(n+m)!} \frac{1}{\mu R^{2n}} \sum_{\nu=1}^{N} \rho_\nu^{2n+1} \beta_\nu f_{nm}^\nu \tag{30}$$

$$\alpha_{H_{nm}} = \frac{(n-m)!}{(n+m)!} \frac{1}{\mu R^{2n}} \sum_{\nu=1}^{N} \rho_\nu^{2n+1} \alpha_\nu h_{nm}^\nu \tag{31}$$

$$\beta_{H_{nm}} = \frac{(n-m)!}{(n+m)!} \frac{1}{\mu R^{2n}} \sum_{\nu=1}^{N} \rho_\nu^{2n+1} \beta_\nu h_{nm}^\nu \tag{32}$$

$$f_{nm}^\nu = \frac{R^n}{\rho_\nu^{n+1}} P_n^m(\sin\theta_\nu) \cos m\lambda_\nu \tag{33}$$

$$h_{nm}^\nu = \frac{R^n}{\rho_\nu^{n+1}} P_n^m(\sin\theta_\nu) \sin m\lambda_\nu \tag{34}$$

$$\delta_n^m = \begin{cases} 1 & \text{if } n = m \\ 0 & \phantom{\text{if }} n \neq m \end{cases} \tag{35}$$

*For details on the eigenfunctions, see the next chapter.

Find the f^{ν}_{nm} and h^{ν}_{nm} as follows. Noting that

$$p_\nu = \frac{R}{\rho_\nu} \tag{36}$$

and that

$$h^{\nu}_{no} = 0 \tag{37}$$

for all values of n, calculate now, separately for each ν, the required f^{ν}_{nm} and h^{ν}_{nm} from the following recurrence relations. To advance in n, evaluate

$$f^{\nu}_{n+1,m} = \frac{p_\nu}{n-m+1}\left[(2n+1)\sin\theta_\nu f^{\nu}_{n,m} - (n+m)p_\nu f^{\nu}_{n-1,m}\right] \tag{38}$$

$$h^{\nu}_{n+1,m} = \frac{p_\nu}{n-m+1}\left[(2n+1)\sin\theta_\nu h^{\nu}_{n,m} - (n+m)p_\nu h^{\nu}_{n-1,m}\right] \tag{39}$$

To advance m, use

$$f^{\nu}_{n+1,n+1} = (2n+1)p_\nu\left[\cos\theta_\nu \cos\lambda_\nu f^{\nu}_{n,n} - \cos\theta_\nu \sin\lambda_\nu h^{\nu}_{n,n}\right] \tag{40}$$

$$h^{\nu}_{n+1,n+1} = (2n+1)p_\nu\left[\cos\theta_\nu \cos\lambda_\nu h^{\nu}_{n,n} + \cos\theta_\nu \sin\lambda_\nu f^{\nu}_{n,n}\right] \tag{41}$$

Start these recurrences from

$$f^{\nu}_{0,0} = \frac{1}{\rho_\nu} \tag{42}$$

$$f^{\nu}_{1,0} = R\frac{\sin\theta_\nu}{\rho^2_\nu} \tag{43}$$

$$h^{\nu}_{0,0} = h^{\nu}_{1,0} = 0 \tag{44}$$

EIGENFUNCTIONS FOR THE TIDE POTENTIAL

The eigenfunctions for the tidal potential are

$$U_{nm} = \frac{\mu R^n}{r^{n+1}} P_n^m (\sin\theta) \cos m\lambda \tag{45}$$

$$V_{nm} = \frac{\mu R^n}{r^{n+1}} P_n^m (\sin\theta) \sin m\lambda \tag{46}$$

Let $y^{(1)}$, $y^{(2)}$, and $y^{(3)}$ be the earth-fixed Cartesian coordinates (see Figure 3) of the general point ("field point"). Note that

$$r = +\left\{(y^{(1)})^2 + (y^{(2)})^2 + (y^{(3)})^2\right\}^{1/2} \tag{47}$$

Also observe

$$p = \frac{R}{r} \tag{48}$$

$$\sin\psi = \frac{y^{(3)}}{r} \tag{49}$$

and

$$V_{n,0} = 0 \tag{50}$$

for all values of n. Now calculate (numerically) the required U_{nm} and V_{nm} from the following recurrence relations.

To advance in n, evaluate

$$U_{n+1,m} = \frac{p}{n-m+1}\left[(2n+1)\sin\psi U_{n,m} - (n+m)pU_{n-1,m}\right] \tag{51}$$

$$V_{n+1,m} = \frac{p}{n-m+1}\left[(2n+1)\sin\psi V_{n,m} - (n+m)pV_{n-1,m}\right] \tag{52}$$

To advance m, use

$$U_{n+1,n+1} = (2n+1)p\left[\frac{y^{(1)}}{r} U_{n,n} - \frac{y^{(2)}}{r} V_{n,n}\right] \tag{53}$$

$$V_{n+1,n+1} = (2n+1)p\left[\frac{y^{(1)}}{r} V_{n,n} + \frac{y^{(2)}}{r} U_{n,n}\right] \tag{54}$$

Start from

$$U_{0,0} = \frac{\mu}{r} \tag{55}$$

$$U_{1,0} = \mu \frac{Ry^{(3)}}{r^3} \tag{56}$$

$$V_{0,0} = V_{1,0} = 0 \tag{57}$$

A TIDE POTENTIAL COMPUTER ROUTINE

This is the first of the two algorithms mentioned in the introduction. It is intended to be a multipurpose computer program, capable of calculating the Newtonian potential associated with the ocean tide. We propose that it be named *Ocean Tide Potential Routine*. This routine has three segments, each indicated by one of the preceding three chapters.

The first program segment is supposed to be coded from Equations 1 through 25. As already explained, it converts the tidal heights into point masses, complete with expressions for the "gravitational charge" m_{ij} and associated position vector. Observe that the m_{ij} oscillate with the tide (Equation 15). It must never be forgotten that several of the equations apply strictly to the M_2 (semidiurnal lunar) tide. Affected are those equations which contain σ and χ. Also, of course, the tidal amplitudes, ζ_{ij}, and the δ_{ij} are valid for the M_2 tide only. Should it be desired to

calculate the disturbing potential associated with a different ocean tide, a suitable table will have to be obtained from a theory for that tide for the ζ_{ij} and δ_{ij}. Also, the new tide frequency and a new algorithm for the position of the tide generating celestial body will have to be substituted for our present σ and χ.

The second program segment is to be assembled from Equations 26 through 44. It calculates the expansion parameters for the potential. Note that this involves certain vast summations over the individual point masses. Independent of the nature of the application contemplated for the Tide Potential Routine, these summations must be extended over all surface area elements. Neither physical nor mathematical criteria exist which would permit us to make any exceptions. The number N of mass points is equal to the number of area elements (each of the latter being 1 degree square in size) necessary to cover the globe except for the land areas and also excluding a substantial cap centered upon the South pole (as specified by the Ocean Tide Program). N is about equal to 45 000. However, only four summations are involved and these need to be done only once, because neither the tide amplitudes nor the phases depend on time.

The third algorithm segment consists of Equations 45 through 57. This is for the calculation of the eigenfunctions. To avoid a misunderstanding, we emphasize that it is strictly a numerical scheme. No analytical expressions are being generated for the eigenfunctions. Rather, this portion of the Tide Potential Routine starts from values for the field point coordinates and proceeds by numerically evaluating the individual eigenfunctions. Because this is done recursively, all eigenfunctions must actually be evaluated for any particular position vector, up to a limit n_{max} on the order and the rank of the functions, irrespective of the nature of the application. Note that n_{max} is the upper summation limit in Equation 26. It depends, of course, on the particular application.

PERTURBING ACCELERATION

The perturbing acceleration to be inserted into the TERRA equations of motion is the positive gradient of the ocean tide potential. Execute now the following procedure, separately for each time line in the orbit integration.

For each time line, $x^{(1)}$, $x^{(2)}$, and $x^{(3)}$ are the Cartesian components of the satellite position vector in inertial space. Assume that the $x^{(i)}$ are given in terms of

kilometers. Part of the TERRA/CELEST equations of motion is an algorithm which performs the necessary transformations* between the inertial frame,** $x^{(i)}$, and the earth-fixed frame, $y^{(i)}$. Now apply this algorithm to rotate the inertial position vector, $x^{(i)}$, into the corresponding earth-fixed position vector, $y^{(i)}$. Find

$$r = +\left\{(x^{(1)})^2 + (x^{(2)})^2 + (x^{(3)})^2\right\}^{1/2} \tag{58}$$

or

$$r = +\left\{(y^{(1)})^2 + (y^{(2)})^2 + (y^{(3)})^2\right\}^{1/2} \tag{47}$$

as convenient. Calculate the values, associated with the satellite position vector, $y^{(i)}$, of the eigenfunctions U_{nm} and V_{nm}.

Evaluate now the Cartesian components of the perturbing acceleration, in the earth-fixed frame:

$$\frac{\partial\phi}{\partial y^{(1)}} = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} \left(F_{nm}\frac{\partial U_{nm}}{\partial y^{(1)}} + H_{nm}\frac{\partial V_{nm}}{\partial y^{(1)}}\right) \tag{59}$$

$$\frac{\partial\phi}{\partial y^{(2)}} = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} \left(F_{nm}\frac{\partial U_{nm}}{\partial y^{(1)}} + H_{nm}\frac{\partial V_{nm}}{\partial y^{(2)}}\right) \tag{60}$$

$$\frac{\partial\phi}{\partial y^{(3)}} = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} \left(F_{nm}\frac{\partial U_{nm}}{\partial y^{(1)}} + H_{nm}\frac{\partial V_{nm}}{\partial y^{(3)}}\right) \tag{61}$$

where

$$\frac{\partial U_{nm}}{\partial y^{(1)}} = \frac{1}{R}\left(\frac{1}{2}A_{mn}U_{n+1,m-1} - \frac{1}{2}U_{n+1,m+1}\right) \tag{62}$$

---

*References for this transformation are unavailable at present. But the transformation is already part of the TERRA coding and will appear in the formal TERRA documentation.

**That is the frame in which the orbit integration is performed.

$$\frac{\partial U_{nm}}{\partial y^{(2)}} = \frac{1}{R}\left(-\frac{1}{2}A_{mn}V_{n+1,m-1} - \frac{1}{2}V_{n+1,m+1}\right) \tag{63}$$

$$\frac{\partial U_{nm}}{\partial y^{(3)}} = \frac{1}{R}\left[-(n-m+1)U_{n+1,m}\right] \tag{64}$$

$$\frac{\partial V_{nm}}{\partial y^{(1)}} = \frac{1}{R}\left(\frac{1}{2}A_{mn}V_{n+1,m-1} - \frac{1}{2}V_{n+1,m+1}\right) \tag{65}$$

$$\frac{\partial V_{nm}}{\partial y^{(2)}} = \frac{1}{R}\left(\frac{1}{2}A_{mn}U_{n+1,m-1} + \frac{1}{2}U_{n+1,m+1}\right) \tag{66}$$

$$\frac{\partial V_{nm}}{\partial y^{(3)}} = \frac{1}{R}\left[-(n-m+1)V_{n+1,m}\right] \tag{67}$$

and where

$$A_{mn} = (n-m+1)(n-m+2) \tag{68}$$

$$U_{n,-1} = -\frac{(n-1)!}{(n+1)!}U_{n,1} \tag{69}$$

$$V_{n,-1} = +\frac{(n-1)!}{(n+1)!}V_{n,1} \tag{70}$$

To find the Cartesian components of the perturbing acceleration in the inertial frame, rotate the vector $\partial\phi/\partial y^{(i)}$ $(i = 1, 2, 3)$ back into the $x^{(i)}$ frame.

OCEAN TIDES IN THE TERRA EQUATIONS OF MOTION

The Equations 58 through 70 listed in the preceding chapter constitute the second of the two algorithms mentioned in the introduction. It might suitably be named Ocean Tide Disturbing Acceleration.

Formula 26 for the tide potential involves a summation over the index n, to be extended to the summation limit n_{max}. In case the Ocean Tide Potential Routine is employed to conduct the previously mentioned experiment of resynthesizing the tidal bulge, a large number of terms will have to be included in the potential. To be specific, the number of terms will have to be about equal to the number of point masses. The latter number is N. n_{max} can be expected to be a number near to the square root of N. However, when applying the Ocean Tide Potential Routine to the computation of the perturbing acceleration due to the ocean tide, in TERRA, a comparatively small number of terms should be sufficient. Only those terms should then be included in the potential which are significant for the purpose of the orbit computation (short arcs and long arcs of the satellite trajectory each suffer perturbations which may arise from different members of the perturbing potential expansion). We shall not try to identify the useful terms now, a priori. Instead, we have chosen to proceed empirically. As far as their application to TERRA is concerned, we are hereby submitting both algorithms for coding, as they stand, preserving full generality as far as the retention or deletion of any terms is concerned. We are proposing that this latter question be resolved, later on, by exploratory TERRA or CELEST runs.

The reader may have noticed that, directly following Equation 47 in the preceding chapter, we specified that the eigenfunctions be evaluated for the earth-fixed satellite position. It was implied that this should be done by using the recursive scheme for the eigenfunctions outlined in the Ocean Tide Potential Routine. We now propose that the programmer decide whether he wishes to call upon the Ocean Tide Potential Routine every time its use is needed during the execution of the Ocean Tide Disturbing Acceleration routine. Remember that the Ocean Tide Potential Routine is external to TERRA while the Ocean Tide Disturbing Acceleration is part of the TERRA coding. We imagine that the programmer may prefer to duplicate the scheme for calculating the eigenfunctions and make it a part of the coding of the Ocean Tide Disturbing Acceleration. That would make the ocean tide term an entirely self-contained part of TERRA, analytically. Of course, the expansion coefficients would still have to be obtained from the Ocean Tide Potential Routine preprocessor.

Finally, one more item remains to be discussed which is related to computational economy. This concerns the time dependent factors

$$\begin{matrix}\cos\\ \sin\end{matrix}\,(\sigma t^* + \chi)$$

occurring in the expansion coefficients for the tide potential. Because these terms are present in the tide potential, they will also appear in the perturbing acceleration associated with the ocean tide. Consequently, it will be necessary to cope with them during each step of the (numerical) orbit integration. In turn, the frequent need to evaluate these trigonometric expressions can be expected to very adversely affect the time required to perform the orbit integration. We thus propose that it be avoided to calculate these factors separately for each time line. Instead, the following scheme may be followed.

Let t^*_i and t^*_{i+1} be the values of Universal time for which subsequent integration steps (time lines) are to be performed. Update the trigonometric time factors as follows.

$$\cos(\sigma t^*_{i+1} + \chi) = \cos\left\{(\sigma t^*_i + \chi) + \sigma\Delta t^*\right\} = \cos(\sigma t^*_i + \chi)\cos\sigma\Delta t^* - \sin(\sigma t^*_i + \chi)\sin\sigma\Delta t^* \qquad (71)$$

$$\sin(\sigma t^*_{i+1} + \chi) = \sin(\sigma t^*_i + \chi)\cos\sigma\Delta t^* + \cos(\sigma t^*_i + \chi)\sin\sigma\Delta t^* \qquad (72)$$

$$\Delta t^* = t^*_{i+1} - t^*_i \qquad (73)$$

Using the algorithm for χ (Equations 18 through 25), update χ whenever t^* enters the following day.

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APPENDIX A

NOTES ON THE TIDE POTENTIAL

NOTES ON THE TIDE POTENTIAL

Two additional comments may be useful in connection with the tide potential. The first concerns some details of the derivation. The second deals with the frequently mentioned recurrence relations for the expansion coefficients and eigenfunctions.

When deriving the tide potential, we started from the potential function for the individual point mass,

$$\phi_\nu = \frac{Gm_\nu}{|\bar{r} - \bar{\rho}_\nu|} \tag{74}$$

where $\bar{r}$ is the position vector of the general point; $\bar{\rho}_\nu$ indicates the position of the point mass. Let

$$\mu_\nu = Gm_\nu \tag{75}$$

Let r, θ, and λ be the geocentric distance, geocentric latitude, and longitude associated with $\bar{r}$. Let ρ_ν, θ_ν, and λ_ν be the geocentric distance, geocentric latitude, and longitude of m_ν. Further, let γ_ν be the angle between $\bar{r}$ and $\bar{\rho}_\nu$. Now,

$$|\bar{r} - \bar{\rho}_\nu| = +\sqrt{r^2 + \rho_\nu^2 - 2r\rho_\nu \cos\gamma_\nu} \tag{76}$$

$$\cos\gamma_\nu = \sin\theta \sin\theta_\nu + \cos\theta \cos\theta_\nu \cos(\lambda - \lambda_\nu) \tag{77}$$

From Page 386 of Reference 13,

$$\frac{1}{|\bar{r} - \bar{\rho}_\nu|} = \sum_{n=0}^{\infty} \frac{\rho_\nu^n}{r^{n+1}} P_n(\cos\gamma_\nu) \tag{78}$$

and from Page 115 of Reference 14,

$$P_n(\cos \gamma_\nu) = P_n(\sin \theta) P_n(\sin \theta_\nu)$$
$$+ 2 \sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} P_n^m(\sin \theta) P_n^m(\sin \theta_\nu) \cos m(\lambda - \lambda_\nu) \tag{79}$$

Now,

$$\phi_\nu = \mu_\nu \sum_{n=0}^{\infty} \frac{\rho_\nu^n}{r^{n+1}} \left\{ P_n(\sin \theta) P_n(\sin \theta_\nu) \right.$$
$$\left. + 2 \sum_{m=1}^{n} \left[\frac{(n-m)!}{(n+m)!} P_n^m(\sin \theta) P_n^m(\sin \theta_\nu) \cos m(\lambda - \lambda_\nu) \right] \right\} \tag{80}$$

At this point, we separated the potential tide into products of terms which depend on $\bar{r}$ only and terms which contain the point mass coordinates only:

$$\phi_\nu = \mu_\nu \sum_{n=0}^{\infty} \rho_\nu^n P_n(\sin \theta_\nu) \frac{P_n(\sin \theta)}{r^{n+1}} + 2\mu_\nu \sum_{n=0}^{\infty} \sum_{m=1}^{n} \underbrace{(\cdots\cdots)}_{\substack{\text{terms which depend} \\ \text{only on source point} \\ \text{coordinates: } \bar{\rho}_\nu}} \times \underbrace{(\cdots\cdots)}_{\substack{\text{terms which contain} \\ \text{only coordinates of the} \\ \text{general point: } \bar{r}}} \tag{81}$$

From here results the following sequence of definitions and equations.

$$\phi_\nu = \phi_{\nu 1} + \phi_{\nu 2} + \phi_{\nu 3} \tag{82}$$

$$\phi_{\nu 1} = \mu_\nu \sum_{n=0}^{\infty} \rho_\nu^n P_n(\sin \theta_\nu) \frac{P_n(\sin \theta)}{r^{n+1}} \tag{83}$$

$$\phi_{\nu 2} = 2\mu_\nu \sum_{n=0}^{\infty} \sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} \rho_\nu^n P_n^m(\sin \theta_\nu) \cos m\lambda_\nu \left\{ \frac{P_n^m(\sin \theta)}{r^{n+1}} \cos m\lambda \right\} \tag{84}$$

$$\phi_{\nu 3} = 2\mu_\nu \sum_{n=0}^{\infty} \sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} \rho_\nu^n P_n^m(\sin \theta_\nu) \sin m\lambda_\nu \left\{ \frac{P_n^m(\sin \theta)}{r^{n+1}} \sin m\lambda \right\} \tag{85}$$

$$\phi_{\nu 1} = \sum_{n=0}^{\infty} E_n^\nu W_n \tag{86}$$

$$\phi_{\nu 2} = \sum_{n=0}^{\infty} \sum_{m=1}^{n} F^{\nu}_{nm} U_{nm} \tag{87}$$

$$\phi_{\nu 3} = \sum_{n=0}^{\infty} \sum_{m=1}^{n} H^{\nu}_{nm} V_{nm} \tag{88}$$

$$E^{\nu}_{n} = \frac{\mu_{\nu}}{\mu} \frac{\rho^{n}_{\nu}}{R^{n}} P_{n}(\sin\theta_{\nu}) \tag{89}$$

$$F^{\nu}_{nm} = 2\frac{\mu_{\nu}}{\mu} \frac{(n-m)!}{(n+m)!} \frac{\rho^{n}_{\nu}}{R^{n}} P^{m}_{n}(\sin\theta_{\nu}) \cos m\lambda_{\nu} \tag{90}$$

$$H^{\nu}_{nm} = 2\frac{\mu_{\nu}}{\mu} \frac{(n-m)!}{(n+m)!} \frac{\rho^{n}_{\nu}}{R^{n}} P^{m}_{n}(\sin\theta_{\nu}) \sin m\lambda_{\nu} \tag{91}$$

$$W_{n} = \frac{\mu R^{n}}{r^{n+1}} P_{n}(\sin\theta) \tag{92}$$

$$U_{nm} = \frac{\mu R^{n}}{r^{n+1}} P^{m}_{n}(\sin\theta) \cos m\lambda \tag{93}$$

$$V_{nm} = \frac{\mu R^{n}}{r^{n+1}} P^{m}_{n}(\sin\theta) \sin m\lambda \tag{94}$$

$$\phi_{\nu} = \sum_{n=0}^{\infty} E^{\nu}_{n} W_{n} + \sum_{n=0}^{\infty} \sum_{m=1}^{n} F^{\nu}_{nm} U_{nm} + \sum_{n=0}^{\infty} \sum_{m=1}^{n} H^{\nu}_{nm} V_{nm} \tag{95}$$

Now, because

$$V_{n0} \equiv 0 \tag{96}$$

as

$$(\sin m\lambda)_{m=0} \equiv 0 \tag{97}$$

there results

$$\sum_{n=0}^{\infty} \sum_{m=1}^{n} H^{\nu}_{nm} V_{nm} \equiv \sum_{n=0}^{\infty} \sum_{m=0}^{n} H^{\nu}_{nm} V_{nm} \tag{98}$$

Now, redefine

$$F^{\nu}_{nm} \rightarrow F^{\nu}_{nm} = (2 - \delta^{0}_{m}) \frac{\mu_{\nu}}{\mu} \frac{(n-m)!}{(n+m)!} \frac{\rho^{n}_{\nu}}{R^{n}} P^{m}_{n} (\sin \theta_{\nu}) \cos m\lambda_{\nu} \tag{99}$$

we have

$$\sum_{n=0}^{\infty} E^{\nu}_{n} W_{n} + \sum_{n=0}^{\infty} \sum_{m=1}^{n} F^{\nu}_{nm} U_{nm} \equiv \sum_{n=0}^{\infty} \sum_{m=0}^{n} F^{\nu}_{nm} U_{nm} \tag{100}$$

Finally, remember that

$$\phi = \sum_{\nu=1}^{N} \phi_{\nu} \tag{101}$$

and also consider the fact that whenever we extended the summation over n to infinity, we actually thought in terms of a finite sum, extended to a finite number, n_{max}. There results now the following collection of equations, for the tidal potential:

$$\phi = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} (F_{nm} U_{nm} + H_{nm} V_{nm}) \tag{102}$$

$$F_{nm} = \sum_{\nu=1}^{N} F^{\nu}_{nm} \tag{103}$$

$$H_{nm} = \sum_{\nu=1}^{N} H^{\nu}_{nm} \tag{104}$$

$$F^{\nu}_{nm} = (2 - \delta^{0}_{m}) \frac{\mu_{\nu}}{\mu} \frac{(n-m)!}{(n+m)!} \frac{\rho^{n}_{\nu}}{R^{n}} P^{m}_{n}(\sin\theta_{\nu}) \cos m\lambda_{\nu} \tag{105}$$

$$H^{\nu}_{nm} = \frac{\mu_{\nu}}{\mu} \frac{(n-m)!}{(n+m)!} \frac{\rho^{n}_{\nu}}{R^{n}} P^{m}_{n}(\sin\theta_{\nu}) \sin m\lambda_{\nu} \tag{106}$$

$$U_{nm} = \frac{\mu R^{n}}{r^{n+1}} P^{m}_{n}(\sin\theta) \cos m\lambda \tag{107}$$

$$V_{nm} = \frac{\mu R^{n}}{r^{n+1}} P^{m}_{n}(\sin\theta) \sin m\lambda \tag{108}$$

Two things are obvious when contemplating Equations 105 and 106. Firstly, according to the Equations 15, 16, 17, and 75, it appears that F^{ν}_{nm} and H^{ν}_{nm} can be written as sums of two terms each of which is a cosine or sine function of a time-dependent argument, multiplied by an expression which depends on point mass position. The same is true for F_{nm} and H_{nm}. Secondly, F^{ν}_{nm} and H^{ν}_{nm} contain the point mass coordinates in a form which suggests that from each a solid harmonic of the latter coordinates should be factored out. This will enable us to apply the recurrence relations which will greatly facilitate the task of evaluating the F^{ν}_{nm} and H^{ν}_{nm}. Both aspects of the matter were reflected when formulating the computer algorithm for the tide potential as listed in the main body of the report.

As far as the recurrence relationships are concerned which we just mentioned and which are also invoked on several occasions in the main body of the report, these reflect the familiar property of the solid harmonics. They have in the past found frequent use in our computer programs for satellite geodesy, in particular on those occasions when we faced the task of evaluating the complicated expressions associated with the earth's gravity field. As they are not novel, they exist in the form of working notes only and were never formally documented.

APPENDIX B

NOTES ON THE ALGORITHM FOR LUNAR POSITION

NOTES ON THE ALGORITHM FOR LUNAR POSITION

Included in the chapter on the tidal point masses is an algorithm for the mean longitude of the moon. This consists of Equations 18 through 25. It specifies values of lunar mean longitude, at 00.00^h UT, for each day during the calendar years 1975, 1976, and 1977. A similar algorithm appears in Reference 4 (see chapter THE MEAN LONGITUDES OF SUN AND MOON and Appendix B). It differs from our present mean longitude routine insofar as it represents the mean longitudes for the sun as well as for the moon, at arbitrary instants of Universal time. Both algorithms are special cases of a more general computer routine which concerns the mean longitudes of sun, moon, and lunar perigee and which was formulated by one of the authors (Groeger) for use with the Schwiderski Ocean Tide Program. No formal documentation is available for the latter routine. But, we expect to publish it sometime in the future in a suitable context.

It may be desirable to state the precise meaning of the term "lunar mean longitude." s (this symbol does not appear elsewhere in this report) is the mean mean longitude of the moon. This is the angle, from the mean equinox of date, measured along the mean ecliptic of date, to the mean ascending node, plus the angle measured from that latter point, along the mean orbit, to the mean perigee, plus the instantaneous value of the mean mean anomaly corresponding to the position of the moon. s is one of the mean elements occurring in the theory of the perturbed motion, relative to the earth, of the moon. As such, it is the secular and the very long periodic portion of the lunar longitude. The latter is also an orbital element in lunar theory. χ as resulting from the present version of the lunar mean longitude algorithm is, as already said above, equal to s, the latter being evaluated at midnight UT.

Our References 15, 16 (especially Pages 537-540), and 17 (Pages 98 and 107) refer to s as a "mean" longitude. This is astronomical usage, adopted for the sake of brevity. According to the above definition of s, the physical item associated with s is actually a mean mean longitude.

Further elaborations related to the mean longitude algorithm may be found in Appendix B of Reference 4. In addition, several figures will be found there which illustrate the concept of the Julian day, for the benefit of those who only occasionally encounter that unit of time.

Finally, our Equation 25 for χ has an accuracy potential of several hundred meters, in the position of the lunar sub point on the earth's surface. We are aware that this is excessively precise for the purposes of present tide models. It was, however, decided to render the polynomial exactly as obtained from Reference 16, because this will anticipate possible further requirements. We do not expect that any worthwhile economies in computer time would result if a truncated polynomial was used.

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